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The Field of a Sound Source Reflected from a Hard Ground

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Abstract: The field of a small (in wavelengths) sound source in the vicinity of a plane, perfectly-reflecting (pressure doubling) surface is investigated in terms of its radiation pattern and its response to the influence of the surface. The radiation impedance is computed as a function of the distance from the surface, and the power radiated by the source is determined from this impedance and the character of the driving mechanism of the source. For a constant-velocity source the radiated power doubles over the value in the unbounded medium as the source approaches the reflector. For a constant pressure source the radiated power approaches zero as the source approaches the reflector. These effects are appreciable only within about 0.25 wavelength of the reflector.

Introduction

We assume an isotropic, monochromatic, sound source at a height H above a plane, perfectly-reflecting (pressure doubling) ground surface and consider the sound pressure amplitude at a distant point, P. Initially we assume that the distance from the ground point beneath the source to P is much greater than H; then the direct ray and reflected ray distances from the source may be considered equal from the standpoint of amplitude but not of phase. At point P the direct and reflected rays will interfere; that is, they may be in phase, or 180° out of phase, or anywhere between, so that the field strength at that point may be twice the value it would have if there were no reflection, or zero, or anywhere between. The extra distance traveled by the reflected ray to get to P causes the interference. Figure 1 shows the geometry of the situation.

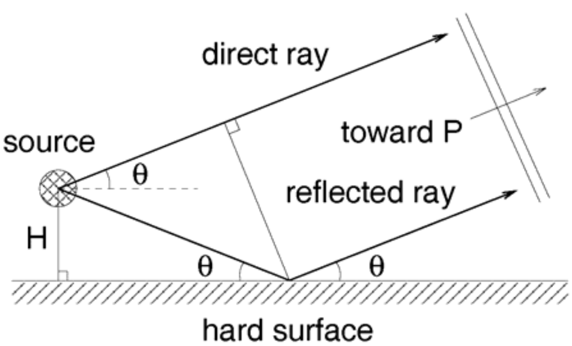

Figure 1. Direct and reflected ray path interference at distances much greater than the source height.

Using the above assumption, computation of the sound field simply consists of the addition of the direct wave from the source to a distant point and the reflected wave to the same point, taking account of the phase delays in the propagation. The far-field direct plus reflected sound pressure, relative to the unbounded (direct only) sound pressure is $(2 + 2\cos\Phi)^{1/2}$, where the phase is $\Phi = (4\pi H/\lambda)\sin\theta$, λ is the wavelength and θ is the elevation angle of a ray from the origin to P.

Further assuming that the source produces a constant volume velocity of sound, we compute the far-field radiation patterns produced by the system at various source heights above the ground. Next, we consider the radiation fields produced by the source when it is excited at a number of frequencies simultaneously, and by a vertical array of sources excited in phase by a number of harmonically related frequencies.

Finally, consideration of the power radiated by the source when it is very close to the reflecting surface leads to a detailed investigation of the radiation impedance seen by the source and the resultant effect on the radiated power.

Source Heights Above One-quarter Wavelength

At source heights above one-quarter wavelength the influence of the reflecting surface on the directly radiated amplitude of the source is small and is neglected for the time being.

Figure 2 is a polar plot of the field strength at a distance from a source one wavelength above a hard ground. The curves show the strength of the field at an arbitrary but constant distance as a function of the angle above the horizon. Note that the strongest signal, twice the amplitude of the direct ray alone, is received at the angles 0°, 30°, and 90° above the horizontal. Between these angles the field strength decreases to zero; at these null angles a distant receiver would hear no sound from our source. Figures 3 and 4 contain the polar diagrams of the radiation fields of sources 0.25, 0.5, 1.5 and 2.0 wavelengths above the hard ground. The higher the source (in wavelengths) is above the ground, the more intricate is the lobe structure of the radiation pattern. In every case, the peak amplitude in any lobe is twice the amplitude of the direct ray alone. *That means that in every case a listener situated on the horizon, or at an angle above the horizon corresponding with the peak of a lobe, will experience a sound level 6.02 dB greater than if the ground were perfectly absorbing; that is, if there were no reflection.* At any other angle the sound level will be less, and if the (squared) amplitude were averaged over all angles, the average level would be 3.01 dB above the value that would obtain if the source were in free space remote from any obstacle.

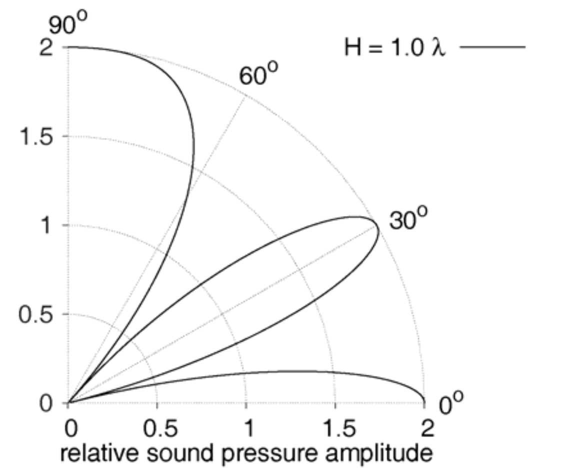

Figure 2. Sound pressure amplitude, normalized to that of an unbounded field, a large distance away from a source one wavelength above a hard ground.

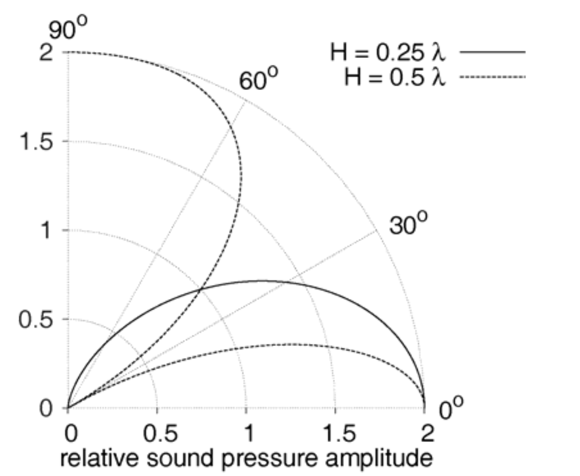

Figure 3. Sound pressure amplitude in the far-field relative to the unbounded field, for sources 0.25 and 0.5 wavelength above hard ground.

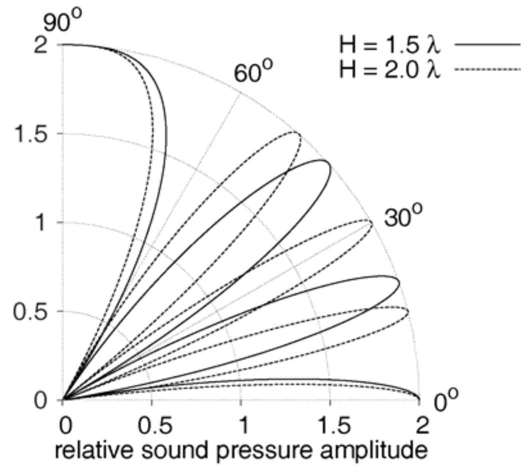

Figure 4. Sound pressure amplitude in the far-field relative to the unbounded field, for sources 1.5 and 2.0 wavelengths above hard ground.

This last statement can be verified by integrating the time-mean-square sound pressure over all solid angles in the upper hemisphere, and normalizing by the same integral for the unbounded field. Doing so, we obtain $2 + 2(\lambda/4\pi H)\sin(4\pi H/\lambda)$. In the limit of large values of H/λ, this expression is identically equal to 2.

We have, of course, assumed an idealized situation. If the ground is partially absorbing, or is absorbing in patches or is otherwise inhomogeneous, the result will be different and would have to be calculated or measured for that specific situation. In no case (except for some pathological cases), however, would the average level above the ground be more than 3.01 dB greater than that which would be obtained were the ground perfectly absorbing. For very low frequencies it is safe to assume many surfaces to be perfectly reflecting.

A Broadband Source

The above arguments were made in the context of a monochromatic source. For a broadband source at a fixed height the resultant field for each individual frequency would be that for the source height in wavelengths corresponding with that frequency. The resultant sound pressure field is the superposition (that is, the linear addition) of the fields for every frequency component in the source. In this more general case, the arguments and the conclusions in the paragraphs above regarding the average dB level and the dB level in the horizontal direction remain the same. For a flat, broad-band spectrum covering at least one octave, the average level for elevated angles would be slightly less than 3.01 dB relative to free space, while at the horizon the maximum level would be 6.02 dB.

Figure 5 is a polar plot showing the pattern produced by eight uniformly-spaced frequency components of equal amplitudes, emitted by a source at a fixed height. The source wave numbers (chosen somewhat arbitrarily) are $k_n = 8\pi n/H$, with n ranging from 1 to 8 in unit steps. Note the lobe on the horizon, and the lower responses at all other elevations. The average sound level over the half-space above the ground is 3.01 dB greater than the free space value.

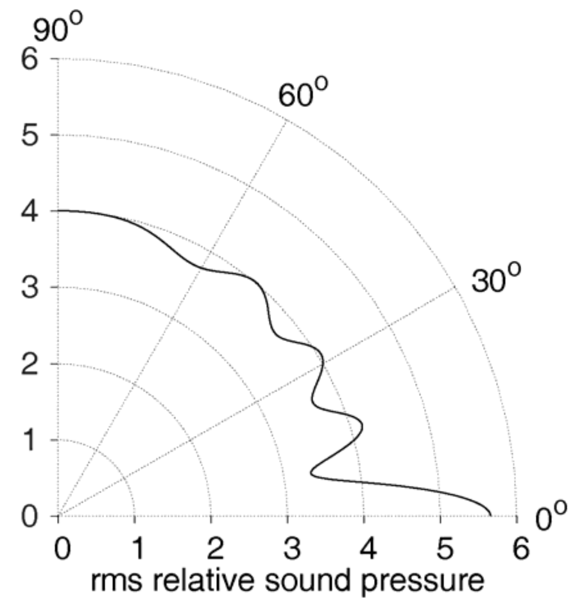

Figure 5. Root-mean-square sound pressure, relative to that of an unbounded single source in the far-field, for a composite source emitting 8 frequencies of equal amplitude. The source wave numbers are chosen to equal k_n = 8 n/H, where n ranges from 1 to 8 in unit steps.

Multiple Sources: the Space/Frequency Equivalence

Suppose one had a vertical array of eight sources, each emitting a distinct frequency corresponding to those of the previous example, and disposed so that their heights were $H_n = H/n$ above the hard ground. The resultant field at a distance then has the same amplitude pattern as that of the single source with height $H = 0.25\lambda$ from Figure 3. Because the field pattern depends solely on the product of wave number and height, one can realize the same pattern by a synthesis either in the frequency domain or in the spatial domain, as convenient. The subject of pattern synthesis has been well developed in the contexts of sonar arrays and of antenna arrays. A possible application for a multiple-frequency, multiple-source array would be an emergency warning system, where it is desired to concentrate the acoustical power in the horizontal direction.

Source Height Less Than One-quarter Wavelength

The above discussion assumed that the presence of the ground plane does not affect the behavior of the source. For calculation of the radiation pattern, this is acceptable. But as the height decreases, the source begins to see an influence from the reflecting plane.

Figure 6 illustrates the fields of single sources 0.1 and 0.02 wavelengths above the ground, respectively. Note that at 0.1 wavelength the depression at the zenith has almost disappeared, while at 0.02 wavelength the pattern is essentially constant at all angles. The integration of these patterns over the hemisphere results in a total radiated power greater than 3.01 dB more than would be radiated into an unbounded space; in fact, approaching 6.02 dB. So far, the results of this study are consistent with the existing literature (Ingard 1957, Maloney 2003). At this point the question of the conservation of energy arises, and it is clear that our simple assumptions about the physical situation must be revised for sources very close to the ground. Implicit in our discussion so far, and of the authors cited above, is the assumption of a constant rms *volume velocity* through some sphere of small radius about the source. This, in turn, requires that the driving energy supply to the source be able to (and constrained to) increase the radiated power as the source approaches the ground. If the power supplied to the source is fixed, the total radiated power in the upper hemisphere cannot be more than that which would be radiated into the total sphere if there were no ground plane.

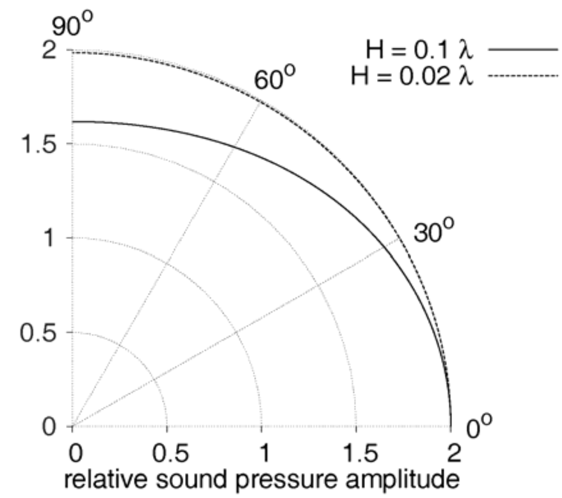

Figure 6. Sound pressure amplitude relative to the unbounded field, a large distance away from sources situated 0.1 wavelength and 0.02 wavelength above a hard boundary.

To examine the effect of ground proximity on the behavior of a sound source it is convenient to adopt the model in which an image source represents the effect of the ground, as shown in Figure 7. The reaction on the source of the reflected wave is represented by the radiation impedance seen by the source. The energy radiated by the source depends on this radiation impedance and also on the internal energy source and the internal impedance of the source, be it mechanical or electrical. The radiation impedance of a spherical source of a given radius is the ratio of the total force applied to its surface to the volume velocity at the surface, simple harmonic motion assumed. Taking the origin of the coordinate system as the center of the source, the pressure radiated by the source toward the surface (or image) varies as $\exp(-jkr)/r$, the reflected wave between the plane surface and the source as $\exp[-jk(2H-r)]/(2H-r)$, and the total pressure as the sum of these two waves. The velocity is proportional to the negative gradient of the pressure (Swenson 1953, Lindsay 1960).

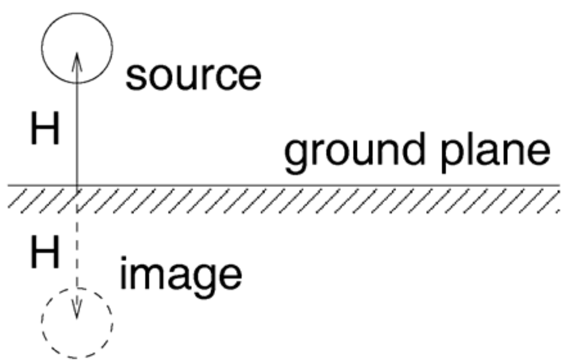

Figure 7. Replacement of the ground plane by an image source.

Choosing $k = 1$ (that is, wavelength=2π) and normalizing with respect to the intrinsic impedance of the unbounded medium, the impedance is:

$$Z(r,H) = \frac{\frac{\exp(-jr)}{r} + \frac{\exp[-j(2H-r)]}{2H-r}}{\left(1-\frac{j}{r}\right)\frac{\exp(-jr)}{r} - \left(1-\frac{j}{2H-r}\right)\frac{\exp[-j(2H-r)]}{2H-r}}$$

In deriving this result, to simplify the mathematics, it was assumed that the pressure is uniform over the surface of a very small (in wavelengths) source. It follows that the particle velocity is everywhere normal to the spherical surface and that the impedance depends only on the radius.

The derivation is valid only for such small sources. Clearly, in the vicinity of an obstacle, the pressure must vary to some extent over the surface of the spherical source. This question has been investigated by a finite element analysis and it is found that for a source of radius 0.05 wavelength, at a distance of 0.25 wavelengths from the reflecting surface, the pressure amplitude on the surface decreases monotonically from front to back by 34 percent. For the same source at 4.0 wavelengths distance, the decrease is 5 percent. As both volume velocity and force are integrals over the same surface, the effect of this variation is minimized but not entirely eliminated. We have therefore chosen to restrict our results to distances greater than 0.075 wavelengths.

Figure 8 shows a plot of the radiation impedance seen by a source of radius 0.05 wavelength at various distances H from the reflecting plane.

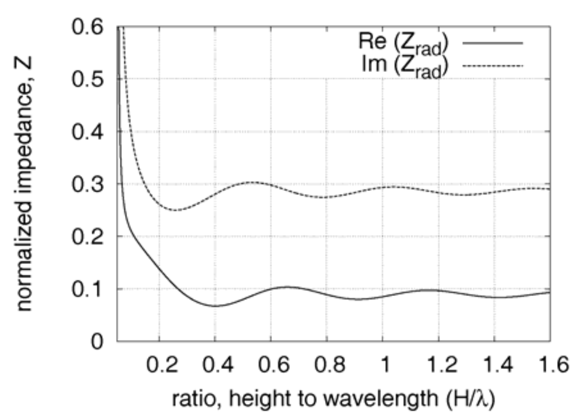

Figure 8. Acoustic radiation impedance of a pulsating sphere, radius 0.05 wavelength, relative to the characteristic impedance, as a function of height near a hard boundary.

At distances greater than about 0.8 wavelength, the impedance depends primarily on the radius r of the source; but as H decreases below 0.25 wavelength, the impedance increases rapidly.

The significance of this result is that at a large distance from the reflecting plane the source is not much affected by the reflector's presence, while close to the plane the reflected wave interacts strongly with the source. A source of acoustical energy must inherently have some internal structure or process; for example, an electromechanical transduction system that converts energy from an electrical or chemical process to a mechanical process which, in turn, drives the surface structure of the source. If this internal process is linear the entire system can be represented for power considerations by a "circuit diagram"' as in electric network theory. Figure 9 represents such a diagram.

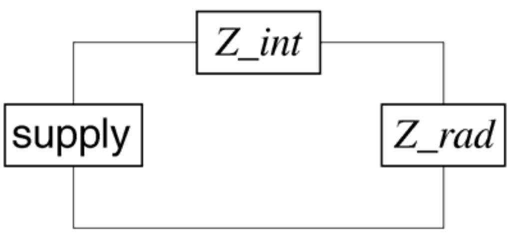

Figure 9. Equivalent circuit representing the source as a supply voltage with internal impedance, coupled to a radiation impedance representing the combined reaction of the air and boundary.

The "supply" in the diagram is assumed to be a constant amplitude (voltage or force, say) supply that is independent of demand. Z_{int} represents the entire internal mechanism of the source; this is justified by Thevenin's Theorem. Z_{rad} is the mechanical radiation impedance, the ratio of the total radial force exerted on the surface of the source to the volume velocity in the medium at the surface. Z_{rad} is obtained directly from Figure 8 as the area of the source is a factor in both the force and the volume velocity. Both "supply" and Z_{int} being independent of source position, the radiated power is the square of the rms volume velocity times the real part of Z_{rad} (this is easily confirmed by dimensional analysis) and thus varies only with H, the distance of the source from the reflecting plane. The power radiated by the source changes little as the source moves, until H decreases below about 0.25 wavelength, after which point it tends toward zero owing to the increase in the radiation impedance.

The decrease is precipitous. As an example, consider a source of zero internal impedance, whose driving force, and therefore whose surface pressure, are constant with source-reflector separation. At 0.075 wavelength, the closest spacing deemed appropriate in view of the assumption of uniform pressure about the source, the radiated power is 0.793 of the power that would be radiated were the reflector absent. If an internal impedance is arbitrarily assumed which is the conjugate of the radiation impedance in an unbounded medium, the result in Figure 10 is obtained. With a higher internal impedance the result will be even more dramatic.

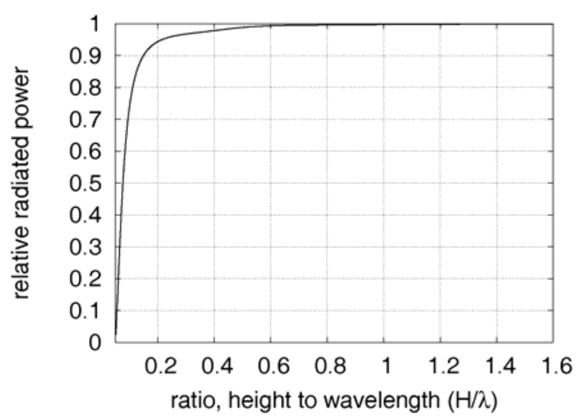

Figure 10. Acoustic power radiated by a pulsating sphere above a hard boundary, relative to the unbounded radiation power. The sphere is driven by a constant (oscillating) supply force having fixed internal impedance. The internal impedance is chosen to be the complex conjugate of the unbounded radiation impedance.

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